Méthodes numériques pour les EDP instationnaires

TP 3: jeudi 14.10.2021
Error measurements

Error measurements

This is a description of the **standard method** to perform measurements of numerical errors. You can use the file `https://www.ljll.math.upmc.fr/despres/BD_fichiers/calcul_error.py` if you want.

1. Implements the upwind scheme

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + a\frac{u_j^n - u_{j-1}^n}{\Delta x} = 0.$$

2. Take a smooth initial data and write the exact analytical solution at time $t = 1$.

3. For a number cell N =10, 20, 40, 80, 160, 320, 640 compute the numerical solution at final time and measure the error between the exact solution and the numerical solution in the L^∞ norm.

   Write the result in a file `res.txt` where the first column is N and the second column is the error.

4. Plot the results in loglog plot.
   With gnuplot:

```
gnuplot
plot res.txt
set logscale
rep
```

   or with matplotlib:

```
import matplotlib.pyplot as plt
plt.loglog(x,y)
```

   You must observe approximatively a straight line.

5. Compare the slope with the theoretical order of convergence: you must get a slope $p = 1$.

6. If the initial data is discontinuous, observe $p = 0$. Explain.

7. Show with numerical experiments that in this case, $p = \frac{1}{2}$ in the L^1 norm and $p = \frac{1}{4}$ in the L^2 norm.

8. Do the same for the Lax-Wendroff scheme.

9. Do the same for the 3 points scheme for the heat equation.

The following example is for the heat equation. One gets (for a convenient exact solution, typically a cosine)

cells	10	20	40	80	160
L^2-error	0.051417	0.012956	0.003247	0.000811	0.000202

This table is represented graphically in the figures below: on the left with usual scales; on the right with logarithmic scales. Additionally the function $x \mapsto x^2$ is displayed to evidence the slope which is the order $p = 2$.